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A Convex Optimization-Based Beamforming Framework for Spectral Efficiency Enhancement in 5G Heterogeneous Massive MIMO Systems

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Abstract

This paper presents a convex optimization-based beamforming framework for enhancing spectral efficiency (SE) in heterogeneous massive multiple-input multiple-output (mMIMO) 5G networks. Although universal frequency reuse improves spectral utilization, it introduces significant multi-cell interference that degrades system performance. To address this challenge, coordinated uplink (UL) and downlink (DL) beamforming schemes are developed to maximize the weighted sum SE under practical constraints. The resulting optimization problem is inherently non-convex and NP-hard; however, by reformulating the signal-to-interference-plus-noise ratio (SINR) and co-channel interference constraints, a tractable convex formulation is obtained. The proposed framework is efficiently solved using CVX, ensuring global optimality. Simulation results demonstrate that the proposed method achieves a minimum SE of 30 bit/s/Hz in the downlink and significantly outperforms benchmark techniques under practical conditions (Kr ≥ 4, N ≥ 12, SNR = 30 dB). Furthermore, increasing the number of users and base station antennas yields substantial additional SE gains, highlighting the scalability of the proposed approach for dense 5G deployments. Simulation results demonstrate that the proposed method achieves a minimum spectral efficiency of 30 bit/s/Hz in the downlink, outperforming existing benchmark schemes under practical conditions (Kr ≥ 4, N ≥ 12, SNR = 30 dB). Furthermore, increasing the number of users and base station antennas leads to significant additional SE gains, highlighting the scalability of the proposed approach for dense 5G deployments.

Keywords: ***beamforming; convex optimization; CVX; massive MIMO; spectral efficiency, 5G***

1. Introduction

The emergence of the fifth-generation wireless technology (5G) has brought about unprecedented demands for high data rates, minimal latency, and pervasive connectivity. As the total user equipment (UEs) in the system and data-intensive applications continues to skyrocket, ensuring efficient use of limited spectral resources has become a critical challenge especially in the mid-band spectrum (C-band), which has been a primary focus for the early 5G deployment worldwide.

Spectral efficiency (bps/Hz), has emerged as a key performance indicator for 5G systems, reflecting the ability to maximize data throughput while minimizing spectral usage. Furthermore, universal frequency reuse, a fundamental aspect of 5G networks where all cells operate on the same frequency band-has posed challenges due to potential multi-cell/inter-cell interference, impacting spectral efficiency (SE) and user experience. To combat this interference, cutting-edge technologies such as massive MIMO (mMIMO) and beamforming

(Bjornson, Bengtsson & Ottersten, 2014; Bengtsson, &Ottersten, 2018) (transmit and receive) have been employed.

Nonetheless, the design of beamformers for a 5G heterogeneous mMIMO system presents a complex challenge, requiring a careful balance between beamforming gain, intra-cell interference, and inter-cell interference. Conventional approaches, such as zero-forcing beamforming (ZFBF) (Quentin & Martin, 2004) and regularized ZFB (RZFB), frequently lead to suboptimal performance and fail to fully maximize the benefits of mMIMO. ZFBF methods are typically sensitive to noise, which can degrade performance, especially in multi-UE environments. Similarly, in the RZFB approach, the selection of the regularization factor is essential in influencing the overall performance of the mMIMO system.

To address these limitations, this paper proposes a convex-optimization-based beamforming framework for 5G heterogeneous massive MIMO systems. Unlike conventional beamforming techniques (Wu & Zhang, 2020; Bazzi & Chafii, 2023a; Bazzi & Chafii, 2023b), the proposed approach reformulates the beamforming design problem into a tractable convex optimization problem, enabling efficient computation of globally optimal solutions. The framework systematically optimizes beamforming vectors to enhance desired signal power while mitigating intra-cell and inter-cell interference. As a result, it achieves improved spectral efficiency in both uplink and downlink transmissions, outperforming existing beamforming strategies in dense multi-user scenarios.

2. Prior Works

Prior studies (Oguejiofor & Zhang, 2016; Oguejiofor, Zhang & Nawaz, 2017: Nwabanne, Oguejiofor & Nnebe, 2023; Oguejiofor, Zhang & Okechukwu, 2024) provide the foundation for this work. Oguejiofor & Zhang (2016) designed non-convex but optimal beamformers for downlink HetNets, maximizing weighted sum-rate via the branch-and-bound algorithm (Joshi, Codreanu & Latva-Aho, 2011). Our present work differs by optimizing weighted sum SE under a broader feasible set. Nwabanne, Oguejiofor & Nnebe (2023) proposed a numerical approach for uplink SE in 5G mMIMO, diverging from analytic methods such as the generalized Rayleigh quotient (Bjornson & Jorswieck, 2013). Unlike these works, we extend beyond single-cell models and optimize both uplink and downlink average SE in a two-tier heterogeneous system.

Beamforming and Convex Optimization

Conventional strategies (e.g., ZFBF [3] and RZFB) suffer from noise sensitivity and dependence on regularization factors. Convex optimization has been used in (Zhang et al., 2022; He et al., 2015) to design energy-efficient beamformers, while Chinnadurai et al. (2018) addressed worst-case weighted sum-rate in multi-cell mMIMO via iterative schemes. Our method convexifies an NP-hard problem by fixing SINR and co-channel interference constraints, enabling tractable optimization.

Weighted Sum-SE Optimization in mMIMO

Works in (Huang et al., 2018; Du et al., 2019; Park, Truong & Nguyen, 2019) explicitly optimize weighted total SE in mMIMO. While their utility functions are comparable to ours, their constraint sets (e.g., per-BS power budgets, interference limits, fairness weights) differ, yielding distinct optima—typically for the downlink. By contrast, we target a two-tier HetNet and obtain coordinated UL/DL beamformers via a convexified formulation with fixed SINR and co-channel interference constraints.

Energy-Efficiency–Oriented Studies

Larsson et al. (2014) advocated mMIMO for next-generation networks due to energy and rate benefits. Björnson, Kountouris & Debbah (2013) minimized power in multi-tier mMIMO; Björnson, Sanguinetti & Debbah (2015) derived closed-form energy-efficiency expressions versus UE count and transmit power. We instead maximize SE and provide numerical (not analytical) results.

Uplink Optimization

Arshad et al. (2020) improved uplink SE in homogeneous two-cell mMIMO using maximum ratio combining (MRC). We generalize to two-tier HetNets and use convex optimization. Tan, Li & Zhou (2022) studied uplink SE/energy efficiency with mixed-ADC architectures; Abebaw et al. (2023) improved uplink multi-cell SE via MMSE and ZF combining. In contrast, our proposed receive beamformers come from convex optimization of an SE utility under convex constraints.

HetNets and Coverage Models

Björnson, Hoydis & Sanguinetti (2017) optimized SE in mMIMO but limited analysis to the coverage tier. Zhang et al. (2025) analyzed achievable rate and energy efficiency in uplink cell-free mMIMO using analytical models. We instead perform numerical SE optimization in a two-tier uplink mMIMO system.

Alternative Approaches

Beyond classical optimization, Siddiqa, Seo & Kim (2025) proposed an SDMA-based network virtualization scheme for IoT-enabled cell-free mMIMO, while Periyathambi and Ravi (2024) applied SSA/CSA for spectrum and energy efficiency in 5G. Our approach directly maximizes SE via convex beamformer design.

Despite extensive research on beamforming and spectral efficiency optimization in massive MIMO systems, existing approaches are largely limited to single-cell models, energy-efficiency objectives, or iterative and analytical solutions that do not fully capture the complexity of multi-cell interference in 5G heterogeneous networks. Moreover, while convex optimization techniques are well-established, their application to uplink and downlink spectral efficiency maximization in multi-tier HetNets remains insufficiently explored.

To address this gap, this work proposes a convex-optimization-based beamforming framework that systematically transforms the inherently non-convex weighted sum spectral efficiency problem into a tractable convex formulation. By incorporating fixed SINR and co-channel interference constraints, the proposed framework enables efficient numerical optimization and achieves superior performance compared to conventional and iterative beamforming methods.

3. Contributions

The main contributions of this work are summarized as follows:

- Optimization Framework: We present a novel framework that transforms non-convex beamforming problems in 5G mMIMO systems into convex formulations, enabling tractable solutions through numerical methods such as CVX.
- Spectral Efficiency Improvement: We demonstrate significant improvements in spectral efficiency for both uplink and downlink scenarios, achieving a theoretical lower bound of 30 bit/s/Hz and outperforming existing techniques under specified conditions.
- Interference Mitigation: We show that the proposed methods effectively suppress multi-cell interference, a critical challenge in 5G networks, thereby enhancing user experience and overall network performance.
- Numerical comparisons: We provide comprehensive numerical simulation results that benchmark the proposed method against traditional beamforming techniques, validating its superiority across various scenarios

4. Materials and Methods

4.1 Materials

This study employs software tools such as CVX, SeDuMi, and MATLAB. CVX is a modeling framework designed for the formulation and solution of disciplined convex programs, including widely used optimization types like linear and quadratic problems. Integrated within MATLAB, CVX effectively transforms the environment into an optimization modeling language and is applied in this research to address constrained disciplined convex optimization tasks. SeDuMi, one of the solvers compatible with CVX, is utilized for executing the underlying optimization computations. This solver implement interior-point methods for efficiently obtaining global optimal solutions to convex problems.

4.2 Methods

In this section, system models for a two-tier heterogeneous 5G network in the downlink (DL) is examined. Mathematical models are formulated to compute the weighted aggregate spectral efficiency of the network, and this spectral efficiency is subsequently optimized—subject to specified constraints—using convex optimization techniques and solved with the CVX tool.

4.3 Downlink System Model

Let's consider the DL of a 2-tier fifth generation HetNet consisting of an aggregate of Kt cells, where Kt denotes the overall number of cells in the system. Additionally, K_p picocells are deployed within the service range of a single macro-cell with K_p representing the overall number of picocells in the system. Since all cells in the HetNet share the same frequency spectrum, inter-cell interference is considered. The set of gNodeBs is denoted as $\mathscr{M} = \{0, \ldots, K_t\}$, where 0 corresponds to the MBS. The j^{th} BS, denoted by BS_j, can be either a macro base station (MBS) or a small cell base station (PBS), and it is considered to be equipped with N antennas, serving K UEs per cell. Each UE is regarded as having a single receive antenna rather than multi-antennas. For practical considerations-such as reduce hardware complexity, a compact form factor, and extended battery life-this work focuses on single-antenna UEs instead of multi-antenna configurations. The collection of UEs served by BS_j is represented as $\subset \{1, \ldots, K_r\}$, where K_r represents the overall number of UEs in the HetNet.

The complex-value baseband data signal $y_k \in \mathbb{C}$ received at UE k from Kt base stations is given by this mathematical expression :

$$y_k = \sum_{j=1}^{K_t} \sqrt{g_{j,k}} \left(\mathbf{h}_{j,k}^{s}\right)^{H} \mathbf{x}_j + z_k. \tag{1}$$

Here $\sqrt{g_{j,k}}$ is the large-scale propagation from the jth base station to UE k. However, in linear scale it is represented as $\sqrt{g_{j,k}} = \frac{\beta\psi}{d_{j,k}^{\eta}}$. Here, β denotes the scaling factor which depends on frequency, environment, and other physical parameters. ψ represents the shadowing component that captures random fluctuations in signal strength caused by obstacles like buildings and trees. $d_{j,k}$ denotes the separation between the transmitter and the receiver while η denotes the pathloss exponent.

Also $\mathbf{h}_{j,k}^{s} \in \mathbb{C}^N$ is the small-scale channel response vector from the jth base station to UE k, while $\mathbf{x}_j \in \mathbb{C}^N$ is the data signal vector transmitted at the jth base station and tailored for it served K active UEs. Additionally, $z_k \in \mathbb{C}$ is the thermal receiver noise with power $\sigma^2 = \mathbb{E}[|z_k|^2]$. This model assumes ideal synchronization, neglecting effects like carrier frequency offset, which are mitigated in practical 5G implementations and can be incorporated in extensions to imperfect CSI scenarios. To achieve the spatial segregation of symbols s_k transmitted to UE $k \in \mathcal{S}j$ from the jth base station, the signal vector being transmitted can be expressed as a linear combination of the data symbols in the following manner:

$$\boldsymbol{x}_j = \sum_{k=1}^{K} \boldsymbol{w}_{j,k} s_k \quad (2)$$

Here, the transmit beamforming vectors associated with each data symbol intended for UE k are denoted by $\boldsymbol{w}_{j,k} \in \mathbb{C}^{N\times 1}$. s_k is believed to be independent and have no correlation, it have power $p = \mathbb{E}[|s_k|^2]$, where p is normalized to unity. Presuming that the BS_l functions as the home BS for UE k, the received complex-baseband signal in Eq. (1) can be given by a different mathematical expression showing the received: desired signal, the intra-cell interference signal and the multi-cell interference signal respectively. This is expressed as

$$y_k = \boldsymbol{h}_{l,k}^H \boldsymbol{w}_{l,k} + \sum_{m\neq k} \boldsymbol{h}_{l,k}^H \boldsymbol{w}_{l,m} + \sum_{j\neq l}^{Kt} \sum_{n\neq k} \boldsymbol{h}_{j,k}^H \boldsymbol{w}_{j,n} + z_k, \quad (3)$$

where $\boldsymbol{h}_{l,k} \triangleq \sqrt{g_{j,k}} \boldsymbol{h}_{j,k}^s$.

Consequently, the signal-to-interference-plus-noise ratio (SINR) at UE k from the j th base station, is expressed as:

$$SINR_{jk} = \frac{\left|\boldsymbol{h}_{l,k}^H \boldsymbol{w}_{l,k}\right|^2}{\sum_{m\neq k} \left|\boldsymbol{h}_{l,k}^H \boldsymbol{w}_{l,m}\right|^2 + \sum_{j\neq l}^{Kt} \sum_{n\neq k} \left|\boldsymbol{h}_{j,k}^H \boldsymbol{w}_{j,n}\right|^2 + \sigma_k{}^2}. \quad (4)$$

Where the numerator in Eq. (4) indicates the desired power of the receive signal, and the first and second terms of the denominator signify the received intra-cell interference power and the received inter-cell interference power respectively, the last term in the numerator represent the noise power.

Spectral efficiency is a function of the $SINR_{jk}$. Lets denote the spectral efficiency of a single UE k in cell j of the HetNet as SE_{jk}, which can be expressed as:

$$SE_{jk} = \log_2\left(1 + SINR_{jk}\right). \quad (5)$$

To determine the total spectral efficiency for all active UEs in cell j, this can be aggregated and the sum spectral efficiency given by:

$$\mathrm{SE_j} = \sum_{\mathrm{k=1}}^{\mathrm{K}} SE_{jk}\,. \quad (6)$$

To account for the different weights of UEs, selected to reflect their respective individual channel gains, we introduce a positive scalar value (weight) v_{jk} for each UE k in cell j. Note that in practical deployments, these weights can be determined through a combination of methods including service legal agreements (SLAs), user classification, and network operator policy.

The weighted sum spectral efficiency for cell j is then represented as:

$$\mathrm{W\_SE_j} = \sum_{\mathrm{k=1}}^{\mathrm{K}} v_{jk} SE_{jk}. \quad (7)$$

Furthermore, the total weighted sum spectral efficiency W_Sum_SE across all cells is given by summing the weighted spectral efficiency of all cells:

$$W\_Sum\_SE = \sum_{j=1}^{K_t} \mathrm{W\_SE_j} = \sum_{\mathrm{j=1}}^{K_t} \sum_{\mathrm{k=1}}^{\mathrm{K}} v_{jk} SE_{jk}\,. \quad (8)$$

A. Downlink Beamforming Optimization

To determine the coordinating beamforming vectors, which are the optimization variables $\{\boldsymbol{w}_{j,k}\}\forall j,k$ that optimize the weighted sum spectral efficiency of the fifth generation mobile communication system ensuring adherence to predefined power limitations and quality of service (QoS) criteria for individual user equipment is a major part of the objectives of this study. Consequently, the mathematical optimization problem formulation is stated as:

$$\underset{\{\boldsymbol{w}_{j,k}\}\forall j,k}{maximize} \quad W\_Sum\_SE \tag{9a}$$

$$\text{Subject to:} \quad \mathrm{SINR_{jk}} \geq \gamma_{\mathrm{jk}} \quad \forall \mathrm{j, k} \tag{9b}$$

$$\sum_{\mathrm{k=1}}^{\mathrm{K}} \mathbf{w}_{\mathrm{j,k}}{}^{\mathrm{H}}\, \mathrm{w}_{\mathrm{j,k}} \leq \mathrm{P_j} \quad \forall \mathrm{j} \in \mathcal{M}, \mathrm{j} \neq 0 \tag{9c}$$

$$\sum_{\mathrm{k=1}}^{\mathrm{K}} \mathbf{w}_{\mathrm{j,k}}{}^{\mathrm{H}}\, \mathrm{w}_{\mathrm{j,k}} \leq \mathrm{P_j} \quad \forall \mathrm{j} = 0. \tag{9d}$$

The objective function in Eq. (9a) represents the system's weighted sum spectral efficiency. The constraints equations (9b~9d) specify the required quality of service limits, which are capped by the threshold parameter γ_{jk} denoted as the SINR limit term for UE k; the PBS power limit, and MBS power limit respectively. Another parameter in the optimization problem is the $\mathrm{P}_{\mathbf{j}}$, which is the power cap at the j th base station.

Given the absence of efficient polynomial-time solutions, the quest to optimize the system's aggregate weighted SE while adhering to the defined constraints (9b ~ 9d) present a formidable optimization hurdle classified as a non-convex and computationally challenging problem. However, Joshi et al. (2011) demonstrated that branch and bound algorithms offer global optimal solutions rather than sub-optimal ones for this type of optimization problem.

Let us investigate every individual constraint function within the maximization problem presented in equations (9b ~ 9d) to uncover the root cause of its inherent non-convexity. The maximization of the objective function in Eq. (9a) relies on the SINR of the system's UEs. Although the transmit power constraint functions in Eqs. (9c ~ 9d) exhibit convex properties, the SINR constraint function in Eq. (9b) is inherently non-convex with respect to the beamformers $\{\mathbf{w}_{\mathrm{j,k}}\}\forall \mathrm{j,k}$, thereby defying classification as either a second-order cone constraint or a semidefinite constraint. To achieve a more profound insight into the non-convex nature of Equations (9a ~9d), let us reformulate it as follows:

$$\underset{\{\boldsymbol{w}_{j,k}\}\forall j,k}{maximize} \quad W\_Sum\_SE \tag{10a}$$

Subject to:

$$\boldsymbol{w}_{l,k}{}^{H}\boldsymbol{Q}_{l,k}\mathbf{w}_{l,k} \geq \gamma_k\,(\Gamma_k) \quad \forall l,\mathrm{k} \tag{10b}$$

$$\sum_{k=1}^{K} w_{j,k}{}^{H}\,\boldsymbol{w}_{j,k} \leq P_j \quad \forall j \in \mathcal{M}, j \neq 0 \tag{10c}$$

$$\sum_{\mathrm{k=1}}^{\mathrm{K}} \mathrm{w}_{\mathrm{j,k}}{}^{\mathrm{H}}\,\mathbf{w}_{\mathrm{j,k}} \leq \mathrm{P_j} \quad \forall \mathrm{j} = 0 \tag{10d}$$

Where: $\Gamma_k = \sigma_k{}^2 + \sum_{m\neq k}\left|\boldsymbol{h}_{l,k}^{H}\boldsymbol{w}_m\right|^2 + \sum_{j\neq l}^{Kt}\sum_{n\neq k}\left|\boldsymbol{h}_{j,k}^{H}\boldsymbol{w}_n\right|^2$, and $\boldsymbol{Q}_{l,k} \triangleq \mathbf{h}_{l,k}\boldsymbol{h}_{l,k}{}^{H}$ is a positive semidefinite matrix $(\mathbf{Q}_{l,k} \geq \mathbf{0})$. Note that the term $\boldsymbol{w}_{l,k}{}^{H}\boldsymbol{Q}_{l,k}\mathbf{w}_{l,k}$ was obtained by expanding the numerator of Eq. (4), $\left|\boldsymbol{h}_{l,k}^{H}\boldsymbol{w}_{l,k}\right|^2$.

Put differently, the non-convex nature of problem (10a ~10d) stems from the SINR constraint of Eq. (10b). This particular constraint becomes non-convex because of the product of γ_k and Γ_k ("intra-cell interference and the multi-cell interference" affecting UE k). To tackle the non-convex nature of the optimization problem, either γ_k *or* Γ_k or both can be set as fixed constants, thereby converting the problem into a recognized convex format. Convexity in non-convex problems arises when reformulation explicitly restricts the search space for the optimal solution particularly when constraint parameters are fixed. This notion is further supported by the principles of convex programming (Boyd & Vandenberghe, 2004: Grant, 2004), which outline how constants, affine expressions, and convex expressions can be formulated to ensure convexity. This reformulation enables the use of optimization software package like CVX (Grant, Boyd & Ye, 2013) to efficiently solve the convex optimization problem in a MatLab environment, ensuring effective resolution of the optimization challenge.

5. The Uplink System Model

The difference between the uplink and downlink systems lies in the direction of communication: uplink transmission refers to signals sent from UEs to their respective base stations, whereas downlink transmission refers to the signal sent from base stations to their respective UEs. For the uplink scenario, a system model similar to that of the downlink is adopted, except for the notable difference that the BS_j is equipped with N antennas with which it observed different fading realizations from any of the K active single-antenna UEs in the system. We assume that the UEs in the system are simultaneously transmitting signals to their respective base stations at the same time and on the frequency, with universal frequency reuse applied. The signal vector $\boldsymbol{y_j}$ received at BS_j from UEs $\in \mathcal{S}_j\ \forall j\ \in \mathcal{M}$ is:

$$\boldsymbol{y}_j = \sum_{k=1}^{K_r} \boldsymbol{h}_{k,j}\, s_{k,j} + \boldsymbol{z}_j. \tag{11}$$

Where $\boldsymbol{h}_{k,j}$ in Eq. (11) means the channel response between UE k to BS_j, whereas $s_{k,j}$ is the information signal transmitted from UE k to BS_j with transmit power $q_{k,j}$. Also, the vector $\boldsymbol{z}_j$ is the additive receiver noise.

For clarity and deeper analysis, Eq. (11) was extended to explicitly show the desired signal, interfering signals (both intra-cell and inter-cell) and the noise. This lead to a new expression of Eq. (11) as

$$\boldsymbol{y}_j = \boldsymbol{h}_{k,j} s_{k,j} + \sum_{\substack{i \neq k \\ i \in \mathcal{S}_j}} \boldsymbol{h}_{i,j} s_{i,j} + \sum_{\substack{p \neq k \\ p \in \mathcal{S}_p}} \boldsymbol{h}_{p,j} s_{p,j} + \boldsymbol{z}_j. \tag{12}$$

To retrieve the desired information signal $s_{k,j}$ received at BS_j, Eq. (12) can be multiplied by a receive beamforming vector $\boldsymbol{u}_{k,j} \in \mathbb{C}^{1\times N}$ yielding a new expression:

$$\widetilde{y}_j = \boldsymbol{u}_{k,j}^H \boldsymbol{y}_j = \boldsymbol{u}_{k,j}^H \boldsymbol{h}_{k,j} s_{k,j} + \sum_{\substack{i \neq k \\ i \in \mathcal{S}_j}} \boldsymbol{u}_{k,j}^H \boldsymbol{h}_{i,j} s_{i,j} + \sum_{\substack{p \neq k \\ p \in \mathcal{S}_p}} \boldsymbol{u}_{k,j}^H \boldsymbol{h}_{p,j} s_{p,j} + \boldsymbol{u}_{k,j}^H \boldsymbol{z}_j. \tag{13}$$

Note that $\boldsymbol{z}_j$, is the noise vector at BS_j , characterize by a zero mean and covariance matrix $\sigma^2 \boldsymbol{I}_N$, with $\boldsymbol{I}_N$ being the N × N identity matrix.

The uplink (UL) signal-to-interference-plus-noise-ratio (SINR) from Eq. (13) is stated as

$$SINR_{k,j}^{UL} = \frac{q_{k,j} \left|\boldsymbol{u}_{k,j}^H \boldsymbol{h}_{k,j}\right|^2}{\sum_{\substack{i \neq k \\ i \in \mathcal{S}_j}} q_{i,j} \left|\boldsymbol{u}_{k,j}^H \boldsymbol{h}_{i,j}\right|^2 + \sum_{\substack{p \neq k \\ p \in \mathcal{S}_p}} q_{p,j} \left|\boldsymbol{u}_{k,j}^H \boldsymbol{h}_{p,j}\right|^2 + \boldsymbol{u}_{k,j}^H \sigma^2 \boldsymbol{I}_N \boldsymbol{u}_{k,j}}. \tag{14}$$

It is important to highlight that the key distinction between the downlink SINR and the uplink SINR as observed when comparing Eq. (4) and Eq. (14), is that in Eq. (4), the downlink interference arises from the beamforming vectors associated with other UEs, whereas in Eq. (14) the uplink interference emanates from the channels of other UEs. This fundamental difference makes it easier to optimize uplink beamforming vectors for each UE individually, unlike in the downlink where optimization must be performed holistically or centrally.

The UL spectral efficiency (SE) achievable by UE $k \in \mathcal{S}_j$ is computed as the base 2 logarithm of Eq. (14) and is mathematically expressed as

$$SE_{k,j}^{UL} = log_2\left(1 + SINR_{k,j}^{UL}\right) \quad (15)$$

A. Uplink Beamforming Optimization

From Eq. (14), it is easier to compute the beamforming vectors that maximize the uplink SINR for each UE, which can be mathematically expressed as

$$\underset{\boldsymbol{u}_{k,j,}\|\boldsymbol{u}_{k,j}\|^2=1}{\operatorname{argmax}} \frac{q_{k,j}\left|\boldsymbol{u}_{k,j}^H \boldsymbol{h}_{k,j}\right|^2}{\sum_{\substack{i\neq k \\ i\in\mathcal{S}_j}} q_{i,j}\left|\boldsymbol{u}_{k,j}^H \boldsymbol{h}_{i,j}\right|^2 + \sum_{\substack{p\neq k \\ p\in\mathcal{S}_p}} q_{p,j}\left|\boldsymbol{u}_{k,j}^H \boldsymbol{h}_{p,j}\right|^2 + \boldsymbol{u}_{k,j}^H \sigma^2 \boldsymbol{I}_N \boldsymbol{u}_{k,j}}, \quad (16)$$

$$\overline{\boldsymbol{u}}_{k,j} = \frac{\left(\sum_{\substack{i\neq k \\ i\in\mathcal{S}_j}} \frac{q_{i,j}}{\sigma^2}\left|\boldsymbol{h}_{i,j}\boldsymbol{h}_{i,j}^H\right|^2 + \sum_{\substack{p\neq k \\ p\in\mathcal{S}_p}} \frac{q_{p,j}}{\sigma^2}\left|\boldsymbol{h}_{p,j}\boldsymbol{h}_{p,j}^H\right|^2 + \boldsymbol{I}_N\right)^{-1} \boldsymbol{h}_{k,j}}{\left\|\left(\sum_{\substack{i\neq k \\ i\in\mathcal{S}_j}} \frac{q_{i,j}}{\sigma^2}\left|\boldsymbol{h}_{i,j}\boldsymbol{h}_{i,j}^H\right|^2 + \sum_{\substack{p\neq k \\ p\in\mathcal{S}_p}} \frac{q_{p,j}}{\sigma^2}\left|\boldsymbol{h}_{p,j}\boldsymbol{h}_{p,j}^H\right|^2 + \boldsymbol{I}_N\right)^{-1} \boldsymbol{h}_{k,j}\right\|_2}. \quad (17)$$

The equation (16) is solved as a generalized Rayleigh quotient (GRQ) which is a seminal approach (Golberg, 1973; Ding & Bent, 2018) yielding Eq. (17). This beamforming vector "minimizes the mean squared error (MSE) between the transmitted signal and the processed received signal". Furthermore, it balances between maximizing the desired signal power and suppressing interfering signal power.

Another classical beamforming scheme that can enhance the uplink spectral efficiency for each UE is the zero forcing method. The beamforming vector for this scheme can be mathematically expressed as (Bjornson &Jorswieck, 2013):

$$\boldsymbol{u}_{k,j} = \left(\sum_{\substack{i\neq k \\ i\in\mathcal{S}_j}} q_{i,j}\left|\boldsymbol{h}_{i,j}\boldsymbol{h}_{i,j}^H\right|^2 + \sum_{\substack{p\neq k \\ p\in\mathcal{S}_p}} q_{p,j}\left|\boldsymbol{h}_{p,j}\boldsymbol{h}_{p,j}^H\right|^2\right)^+ \boldsymbol{h}_{k,j}\,. \quad (18)$$

This is basically the product of the desired signal channel and the pseudoinverse of the interfering channels in the system. It enables the desired signal to be projected onto the subspace that is orthogonal to the interference existing in the system.

Alternatively, this section demonstrate how the uplink SE of the system can be optimized through the selection of optimal receive beamformers which constitute the solutions to the formulated optimization problem. The problem is expressed in standard form as follows:

$$\underset{\{\boldsymbol{u}_{k,}\}_{k=1}^{K_r}}{minimize} \quad -\sum_{j=1}^{k_t}\sum_{k=1}^{K} v_{k,j} SE_{k,j} \quad (19a)$$

Subject to

$$SINR_{k,j}^{UL} \geq \gamma_{k,j} \quad k = 1, \dots, K; \text{j}= 1, \dots, k_t \quad (19b)$$

Note that formulation of an optimization problem in the standard form typically begin with the keyword "Minimize". But since what we are dealing with is a utility function that should be maximized, we then minimized the additive inverse of the cost function which is the same as maximizing the utility function (weighted aggregate SE of the uplink system) under identical constraints.

From Eq. (19a), we know that the cost function is convex. However, similar to Eqs. (9b ~ 9d), the constraint function in Eq. (19b) is not convex. Therefore, this optimization problem cannot be classified as a convex problem. Classifying an optimization problem is important, as it allows for the use of a suitable numerical algorithm designed for that class of problem. To properly classify the constraint function in Eq. (19b), we rewrite it in a form similar to Eq. (10b), which gives:

$$q_{k,j}\left|\boldsymbol{u}_{k,j}^{H}\boldsymbol{h}_{k,j}\right|^{2} \geq \gamma_{k,j}(\Gamma_{k,j}) \;\; k = 1, \dots, K; \text{j}= 1, \dots, k_t \,. \qquad (20)$$

According to the discipline convex programming (DCP) ruleset Grant (2004), derived from the fundamental tenets of convex analysis, the DCP guideline represent a set of conditions that are adequate for convexity but not obligatory. Based on this ruleset, the no product rule between non-constant expressions as shown on the right-hand side of the inequality expression actually renders Eq. (20) nonconvex. To make it convex. We resolve this by expressing $\Gamma_{k,j}$ (the summation of the interference and noise term) and $\gamma_{k,j}$ (the SINR limit term) as constant expressions or values for all UEs in the system. This lead to a newly reformulated convex optimization problem stated as:

$$\underset{\{\boldsymbol{u}_{k,}\}_{k=1}^{K_r}}{maximize} \quad \textstyle\sum_{j=1}^{k_t}\sum_{k=1}^{K} v_{k,j}SE_{k,j} \qquad (21a)$$

Subject to

$$q_{k,j}\left|\boldsymbol{u}_{k,j}^{H}\boldsymbol{h}_{k,j}\right|^{2} \geq \gamma_{,}(\Gamma) \qquad (21b)$$

This problem can now be resolved by interior point methods; whose general-purpose implementations are available in SeDuMi. However, in this work, we solved the problem using CVX (Grant, Boyd & Ye, 2013), because it enforces the conventions dictated by the DCP ruleset.

Note that CVX can employ different solvers, such as SeDuMi, Gurobi and MOSEK, to solve various optimization problems. For example, when solving a convex optimization problem using CVX with solvers like SeDuMi, it can efficiently find the global optimum for convex problems. In contrast, iterative schemes such as iterative WMMSE and sequential convex approximation (SCA) may attain to a local optimum, depending on the problem structure and initialization. Iterative WMMSE and SCA typically has slightly lower per-iteration computational complexity; however, their overall runtime may increase due to multiple iterations, especially for large-scale systems (e.g., high N or Kr).

6. Numerical Simulation

In this study, we assess the downlink performance of our proposed method by comparing it with Oguejiofor's method (Oguejiofor et al.,2017) as well as various traditional techniques such as the zero-forcing method, sequential convex approximation (SCA) and egoistic beamforming method. This assessment focuses on the average overall SE, mean SNR, and the transmitter antennas array size. Similarly, we evaluate the uplink performance of our presented method against the generalized Rayleigh quotient method (GRQM), the SCA method and the classical zero-forcing method. This evaluation considers the average SE, mean SNR, the number of transmitting antennas, and the numbers of UEs.

6.1 Simulation Settings

Let's consider an Urban-Macro deployment scenario for the 5G New Radio rollout. The selected frequency spectrum falls within the 3.4-4.2 GHz frequency band (C-band), which forms part of the key mid-band spectrum designated for early 5G deployments in many countries worldwide. The carrier frequency bandwidth is presumed to be 100 MHz. SNR is measured as the transmit power limit divide by noise power, with the noise power normalize to 1 unit. The SNR per bit is not directly linked to the sum spectral efficiency here, as the system performance is dominated by interference, and therefore was not considered. The simulation results are averaged over the combined effects of random circularly symmetric complex Gaussian channels, pathloss and shadowing realizations.

In this simulation, at least three PBSs are arbitrarily positioned at high-traffic areas within the Macro cell coverage area. The least separation between Pico sites is 43 meters. Since these sites are not spatially separated, interference among PBSs is considered. Additionally, the Macro base station is positioned at least 82 meters away from any Pico base station.

User Equipment (UEs) in this 5G system are considered to be equally distributed within the cell-edge area, where they experience both multi-cell interference and signal path loss from their serving base station. The UEs served by PBSs follow a uniform distribution between 42 meters and 63 meters from their respective PBSs, whereas the UEs served by the macro base station follow a uniform distributio between 215 meters and 265 meters from the MBS. Furthermore, the separation between MBS UEs and the PBSs varies from 40 to 44 meters, while the distance between PBS UEs and the MBS ranges from 225 to 275 meters.

The remaining system parameters adhere to the 5G-ACIA specifications (Ericsson, 2020) and 3GPP technical specification (3GPP, 2018)). The user equipment transmits at 23 dBm, while the macro base station and the pico base station transmit at 46 dBm and 30 dBm, respectively. Unless otherwise specified, this simulation setup is used throughout

7. Results and Discussion

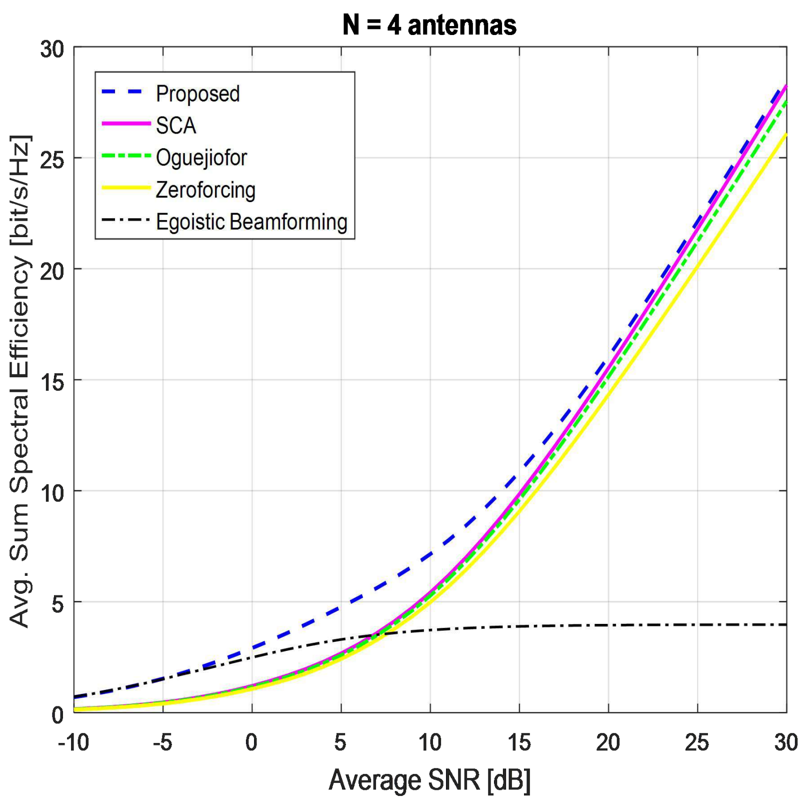

Figure 1. Downlink Avg. Sum SE Versus Avg. SNR for Different Beamforming Schemes, Kr = 4, N = 4

Figure 1 illustrates how the system's average downlink sum spectral efficiency varies with increasing signal-to-noise ratio. The blue dashed curve corresponds to the proposed method, while the magenta dashed curve shows the performance obtained using sequential convex approximation. The yellow curve represents the zero-forcing approach, and the green dashed and black curves depict the results of Oguejiofor's method and the egoistic beamforming technique, respectively. The proposed approach demonstrates superior performance compared with the egoistic beamforming technique by Godana & Gesbert (2013), the zero-forcing beamforming technique (Quentin & Martin, 2004), the approach by Oguejiofor *et al. (2017)*, and the sequential convex approximation (SCA) method (Yu et al., 2022).

The egoistic beamforming method is developed without accounting for interference from neighboring cells, as its primary objective is to construct beamforming vectors that maximize the signal power for user equipment within individual cells, while neglecting inter-cell interference. As evident from the results, this method under-performs in a 5G system, where multi-cell interference plays a significant role due to frequency re-use1 deployment.

The zero-forcing method although effective in nullifying interference, does not account for the noise in the system. But is effective in nullifying interference in the system. Hence, its performance is also inferior compared to the proposed method. Finally, the proposed method achieves comparable or marginally higher SE than the SCA method. The SCA method addresses non-convex optimization problems by iteratively reformulating them into a series of convex sub-problems.

According to 3GPP simulations, the minimum attainable downlink SE for a 5G network is estimated to be 30 bit/s/Hz. Taking this as a baseline, the proposed method requires the following system attribute and parameters (*SNR* = 30 dB, *N* = 4, *Kr* = 4) to achieve it. Under the same parameters, the egoistic beamforming method fails to reach this efficiency, and the zero-forcing method also falls short, while the approaches by Oguejiofor *et al. (2017),* and the SCA method require an increased SNR to attain comparable performance.

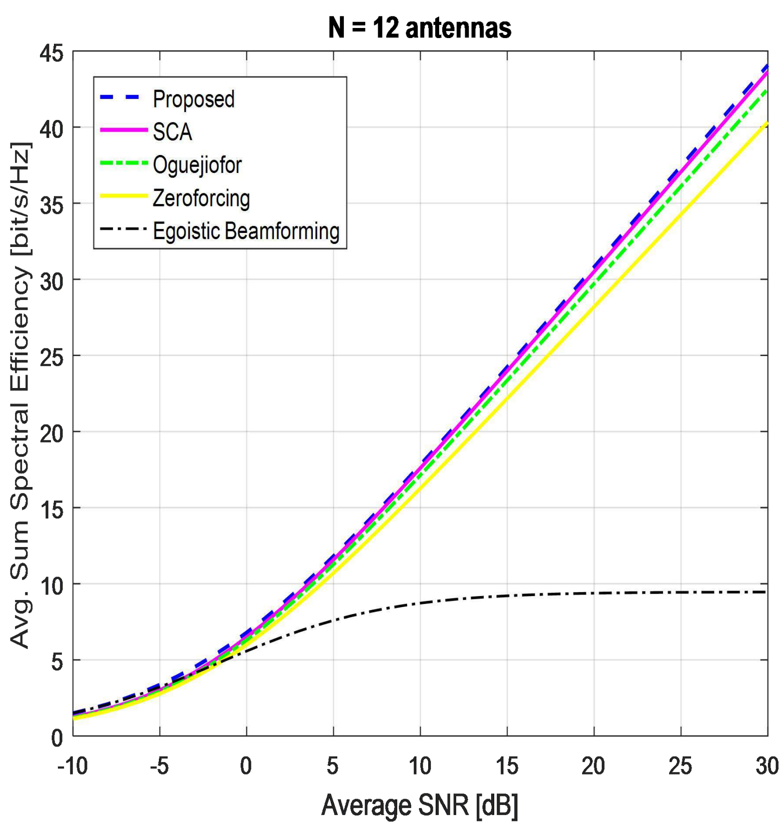

Figure 2. Various Downlink Avg. Sum SEs Versus Avg. SNRs for Kr = 4, N = 12

This Figure 2 mirrors the structure of Figure 1, but differs in that the performance of the presented method—and the other benchmarked techniques listed in Figure 1—is evaluated by increasing the number of antennas increased from $N = 4$ to $N = 12$, while keeping the number of active UEs in the system unchanged.

The proposed method outperformed the other techniques, as illustrated in Figure. 2. At an *SNR* of 30 dB, the presented approach attained an SE = 44 bit/s/Hz, while the SCA method, Oguejiofor *et al.* method and the zero-forcing method also surpassed 30 bits/s/Hz at the same SNR. In contrast, the egoistic beamforming technique reached only 10 bit/s/Hz. Furthermore, a comparison between Figure. 1 and Figure. 2, reveal that the proposed method attained a higher downlink SE of 44 bit/s/Hz at 30 dB SNR, corresponding to an improvement of 14 bit/s/Hz. This gain is attributed to increasing number of transmit antennas from $N = 4$ to $N = 12$ per base station in each cell.

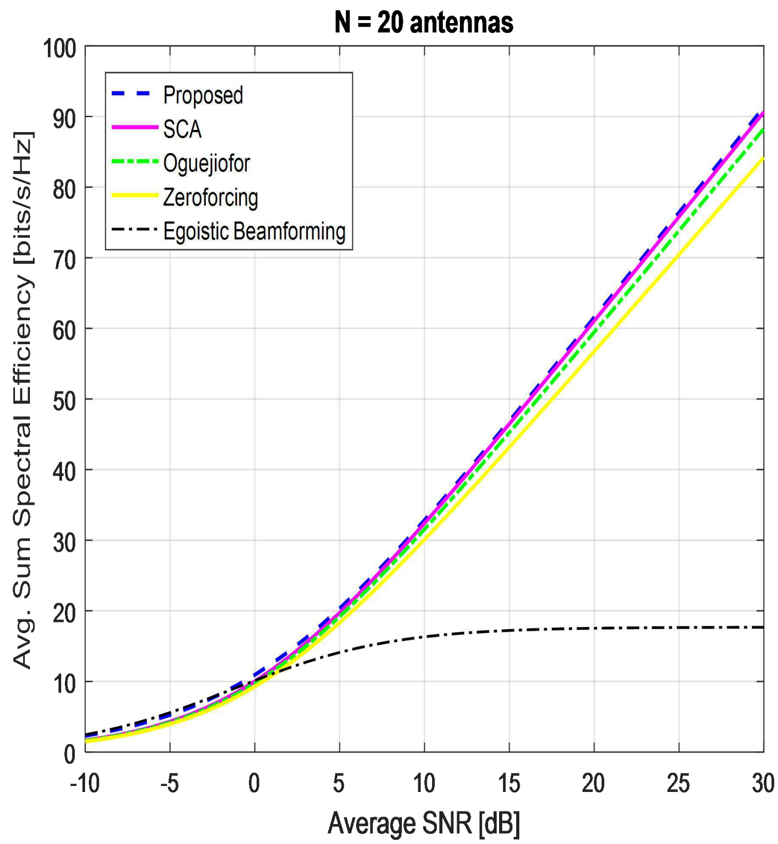

Figure 3. Various Downlink Avg. Sum SE Versus Avg. SNRs for Kr = 9, N = 20

This Figure 3 is analogous to Figure 1, but differs in that the effectiveness of the presented method—and the other comparison techniques listed in Figure 1—is evaluated after increasing the number of antennas from $N = 4$ to $N = 20$. In addition, the number of active UEs in the system is raised from Kr = 4 to Kr = 9.

The presented approach demonstrated better performance compared with the other methods shown in Figure 3. At an SNR of 30 dB, the average total downlink SE achieved with the proposed approach is 92 bit/s/Hz. In contrast, the average total downlink SE in Figure 2 (with parameters $Kr = 4$, $N = 12$) is only 44 bit/s/Hz at the same SNR. As a result, the SE in Figure 3 is 48 bit/s/Hz higher than that in Figure 2, while maintaining the same SNR. This suggests that when coordinated beamforming techniques such as the one introduced in this study are employed, the average overall SE of the system increases as both the number of user equipments and the number of transmit antennas at each base station increases.

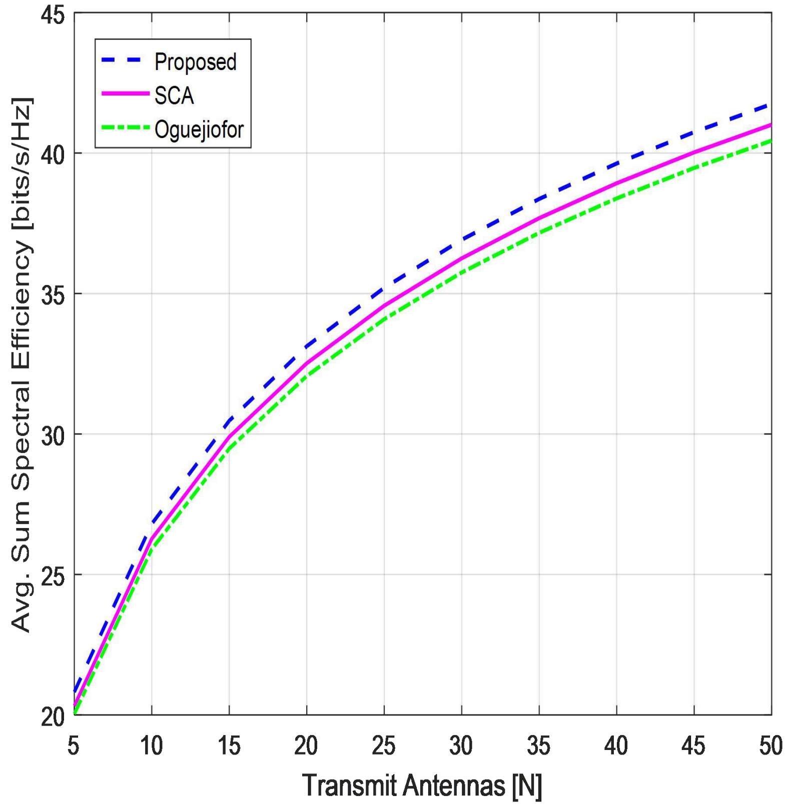

Figure. 4. Downlink Avg. Sum SE Versus Various transmit antennas at SNR = 10 dB

The Figure 4 illustrates how the system's average downlink sum spectral efficiency varies with the increase in the number of transmit antennas at the base station. The blue dashed curve represents the proposed method, while the magenta dashed and green dashed curves show the performance achieved using sequential convex approximation and Oguejiofor's method, respectively**.** In Figure 4, the graph depicting the average overall SE of the system versus various base stations transmit antennas reveals that at a SNR of 10 dB, with 20 transmit antennas, the SE attained by the propose method is approximately 34 bit/s/Hz, while that of the SCA method is about 33 bit/s/Hz, and the method by Oguejiofor *et al.(2017)* is around 32 bit/s/Hz. These values are fairly similar to those presented in Figure 3 at a SNR of 10dB. However, it is demonstrated that to meet the minimum downlink SE of 30 bit/s/Hz required by 3GPP for a 5G system, the base station transmit antennas must exceed 10 when the SNR is 10dB.

7.2 Uplink Results

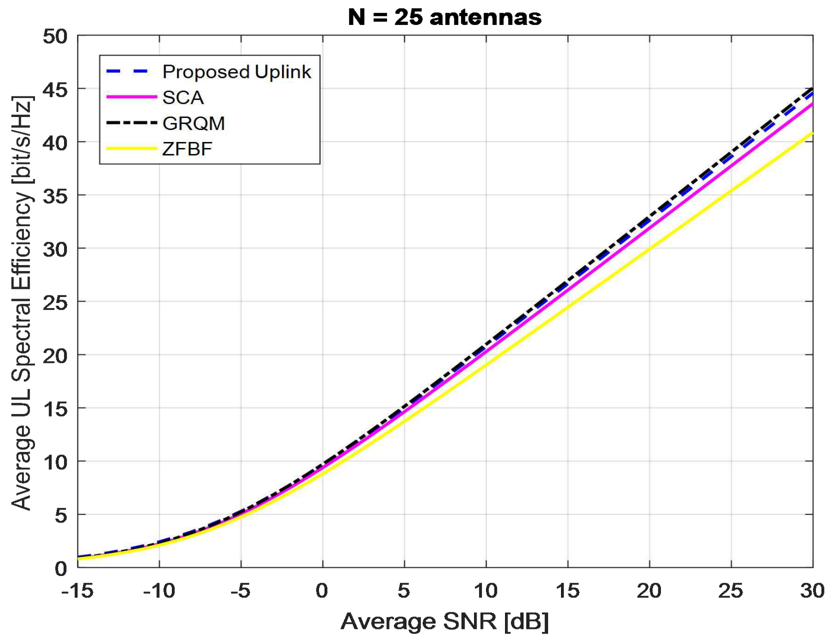

Figure 5. Avg. Uplink SEs Versus Various SNR levels for Kr = 4, N = 25

The Figure 5 depicts how the system's average uplink sum spectral efficiency changes with increasing average signal-to-noise ratio. The blue dashed curve corresponds to the proposed method, while the magenta dashed curve shows the performance obtained using sequential convex approximation. Additionally, the yellow curve represents the zero-forcing method, and the black dashed curve illustrates the performance achieved using the Generalized Rayleigh Quotient method.

In Figure 5, the average uplink spectral efficiency achievable at high SNR of 30 dB, with N = 25 receive antennas and Kr = 4 UEs, is 45 bit/s/Hz for the presented method, 44 bit/s/Hz for the SCA method, 41 bit/s/Hz for the zero forcing method, and 45.5 bit/s/Hz for the Generalized Rayleigh Quotient Method (GRQM). While the proposed method outperforms the zero-forcing method at both high and low SNRs, the SCA method also shows modest improvement over zero forcing method but remains slightly inferior to the proposed method. The GRQM demonstrates a slight advantage at high SNR in comparison to the proposed method; however, at low SNR (- 20 dB) to moderate SNR (5 dB), both methods yield the same performance.

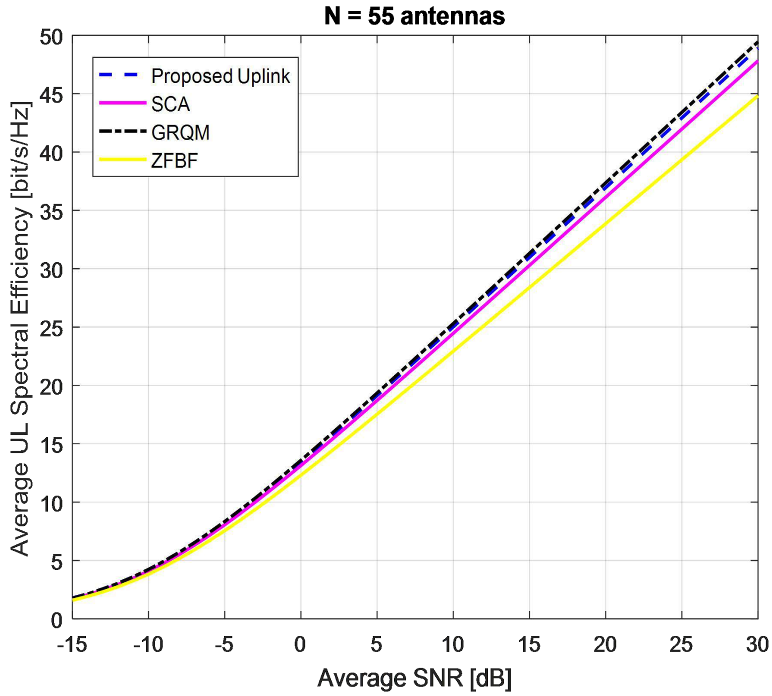

Figure 6. Avg. Uplink SEs Versus Various SNR levels for Kr = 4, N = 55

This Figure 6 follows the same structure as Figure 5, but differs in that the effectiveness of the presented method—and the other comparison techniques illustrated in Figure 5—is evaluated with the number of antennas increased from $N = 25$ to $N = 55$, while keeping the number of active UEs in the system unchanged.

In Figure 6, the average uplink spectral efficiency achievable at high SNR of 30 dB, with N = 55 receive antennas and Kr = 4 UEs, is 48.5 bit/s/Hz for the proposed method, 47.5 bit/s/Hz for the SCA method, 45 bit/s/Hz for the zero-forcing method, and 49 bit/s/Hz for the Generalized Rayleigh Quotient Method (GRQM). While the proposed method outperforms the zero forcing method at both high and low SNRs, the SCA method shows modest improvement over zero forcing method as well but remain slightly inferior to the proposed method. The GRQM demonstrate a slight advantage at high SNR compared with the proposed method; however, at low SNR (- 20 dB) to moderate SNR (5 dB), both methods yield the same performance.

Observations from Figures. 5 and 6 show that as the base station antennas number increases from 25 to 55, with other system parameters held constant, the average uplink spectral efficiency improves from 45 bit/s/Hz to 48.5 bit/s/Hz for the proposed method. This corresponds to a gain of 3.5 bit/s/Hz in the average uplink spectral efficiency.

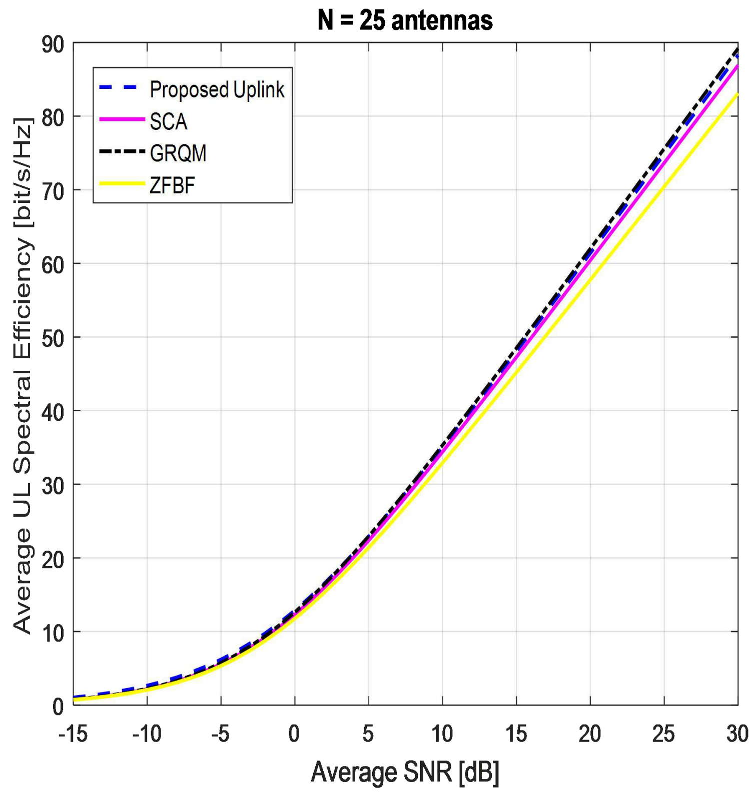

Figure 7. Avg. Uplink SEs Versus Various SNR levels for Kr = 9, N = 25

This Figure 7 resembles Figure 5, but differs in that the effectiveness of the presented method—and the other comparison techniques presented in Figure 5—is analyzed with the number of active UEs increased from $Kr = 4$ to $Kr = 9$, while the base station antenna number remains unchanged.

In Fig. 7, the average uplink SE attainable at high SNR of 30 dB, with $N = 25$ receive antennas and Kr = 9 UEs, is 89 bit/s/Hz for the proposed method, 88 bit/s/Hz for the SCA method, 83 bit/s/Hz for the zero-forcing method, and 90 bit/s/Hz for the Generalized Rayleigh Quotient Method (GRQM). While the proposed method outperforms the zero-forcing method at both high and low SNRs, the SCA method also shows modest improvement over zero forcing but remain slightly inferior to the proposed method. The GRQM demonstrates a slight advantage at high SNR compared with the proposed method; however, at low SNR of (- 15 dB to - 5 dB), the proposed method slightly outperforms GRQM.

Observations from Figures. 5 and 7 show that as the active UEs number increases from 4 to 9, with other system parameters held constant, the average uplink spectral efficiency rises from 45 bit/s/Hz to 89 bit/s/Hz for the proposed method. This corresponds to the gain of 44 bit/s/Hz in the average uplink SE.

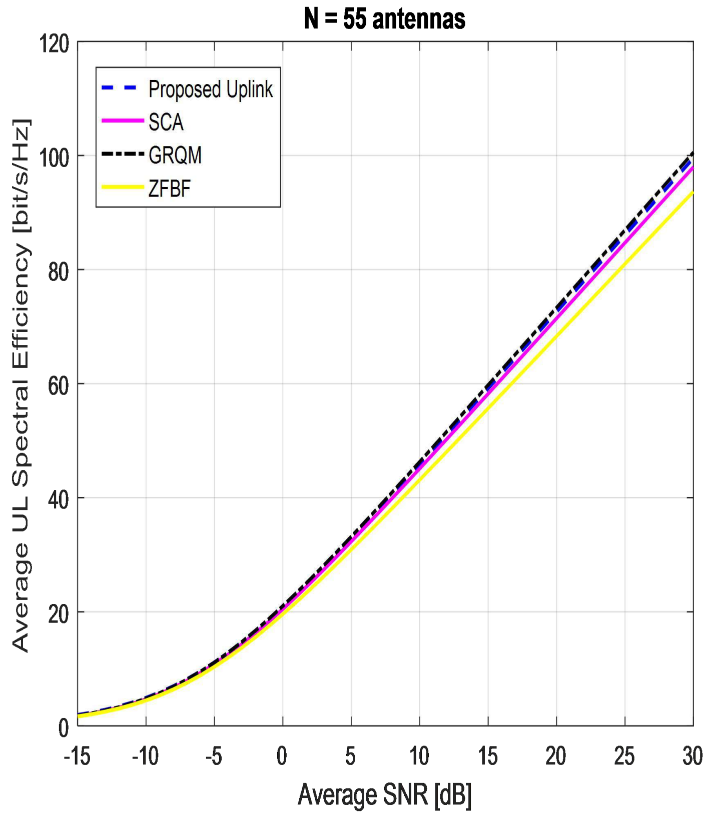

Figure 8. Avg. Uplink SEs Versus Various SNR levels for Kr = 9, N = 55

This Figure 8 mirrors Figure 5, but differs in two key aspects: the number of active UEs is increased from *Kr* = 4 to *Kr* = 9, and the base stations antennas number is raised from *N* = 25 to *N* = 55. The effectiveness of the presented method and the other comparison techniques from Figure 5 is evaluated under these updated conditions.

In Figure 8, the average uplink SE attainable at high SNR of 30 dB, with *N* = 55 receive antennas and *Kr* = 9 UEs, is 100 bit/s/Hz for the proposed method, 99 bit/s/Hz for the SCA method, 93 bit/s/Hz for the zero-forcing method and 100.5 bit/s/Hz for the Generalized Rayleigh Quotient Method (GRQM). While the proposed method outperforms the zero-forcing method at both high and low SNR, the SCA method also shows modest improvement over zero forcing but remains slightly inferior to the proposed method. The GRQM demonstrates a slight improvement at high SNR; however, at low SNRs (- 15 dB to - 5 dB), both methods yield the same performance.

Observations from Figures. 5 and 18 show that as the active UEs number increases from *Kr* = 4 to *Kr* = 9, and the base station antennas number increases from *N* = 25 to *N* = 55 (with other system parameters held constant), the average uplink spectral efficiency rises from 45 bit/s/Hz to 100 bit/s/Hz for the proposed method. This corresponds to an improvement of 55 bit/s/Hz in the average uplink SE.

Overall, the observed performance gains confirm that the proposed convex optimization framework effectively balances signal enhancement and interference suppression in multi-cell environments. The consistent improvement across varying antenna configurations and user densities demonstrates the robustness of the approach. Furthermore, unlike conventional methods such as zero-forcing and SCA, which either neglect noise effects or rely on iterative approximations, the proposed method achieves globally optimal solutions under the convexified formulation. These findings validate the suitability of the proposed framework for practical 5G heterogeneous deployments, particularly in interference-limited scenarios.

8. Conclusion

This study addressed the challenge of enhancing spectral efficiency in fifth-generation heterogeneous massive MIMO systems by proposing a convex optimization-based beamforming framework. Unlike conventional beamforming techniques, which often yield suboptimal solutions and exhibit sensitivity to interference, the proposed framework systematically transforms the weighted sum spectral efficiency maximization problem into a tractable convex formulation. This enables the efficient computation of globally optimal beamforming vectors using CVX.

The proposed approach demonstrates superior performance compared to traditional methods such as zero-forcing and sequential convex approximation, particularly in interference-limited multi-cell scenarios. Notably, a minimum spectral efficiency of 30 bit/s/Hz is achieved under practical system conditions, validating the effectiveness of the framework. Furthermore, the results indicate that the method scales efficiently with increasing numbers of antennas and users, making it well-suited for dense 5G deployments.

Overall, this work establishes a robust and scalable beamforming design framework that effectively mitigates multi-cell interference while significantly enhancing both uplink and downlink spectral efficiency. The proposed convex optimization approach provides a solid foundation for the development of next-generation high-capacity wireless networks. Future research may extend this framework to scenarios with imperfect channel state information, dynamic user mobility, and real-time adaptive implementations.

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Conflict of Interest
The authors declare no conflict of interest.

Ethics Approval
No human participants or animals were involved. The study followed institutional safety protocols.

Data and Code Availability
The data and code that supports the findings of the study are available from the corresponding author, Obinna Samuel Oguejiofor, upon reasonable request upon reasonable request.

Author Contributions (CRediT)
Conceptualization: O.S.O; Methodology: O.S.O.; S.N.U.; Investigation: S.N.U.; Validation: S.N.U.; M.A.; Writing—original draft: O.S.O.; Writing—review & editing: S.N.U; M.A.

AI-Use Disclosure
The authors used a generative language tool for grammar polishing only; all technical content and conclusions were authored and verified by the authors.

Notations: The following notation is used in this paper

$(\cdot)^H$ — Transpose-Conjugate operator
$(\cdot)^T$— Transpose operator
$|\cdot|$ — Magnitude of convex variable
$\|\cdot\|_2$ — L2 norm of a vector
$\mathbb{E}\{\cdot\}$ — The average value taken over a random variable
K — The overall number of UEs in a cell
BSj — jth base station

N — Number of antennas

$\mathcal{S}$j — The collection of UEs served by BS_j
$\sqrt{\mathrm{g_{j,k}}}$ — The Large-Scale pathloss from BS_j to UE k
$\mathbf{h}_{\mathrm{j,k}}^{\mathrm{s}}$ — The Small-scale fading channel from BS_j to UE k
$\boldsymbol{w}_{j,k}$ — Transmit beamforming vector meant for UE k from BS_j
$\boldsymbol{h}_{l,k}^{H}$ — Channel response from BS_l to UE k

$\boldsymbol{u}_{k,j}$ — Receive beamforming vector

γ_k— SINR limit term at UE k

Γ_k — Interference limit term to UE k (Inter-cell +Multi-cell)

$\mathcal{M}$ — The set of gNodeBs

Kr — Overall number of active UEs in the system
$q_{k,j}$ — Uplink transmit power from UE k to BS_j

σ^2 — Noise power

Kp — Overall number of Pico cells in the system
Kt — Overall number of cells in the system
$\mathbf{Q}_{l,k} \geq \mathbf{0}$ — Positive Semidefinite Matrix
$P_{\boldsymbol{j}}$ — Power cap at the BS_j

$\boldsymbol{X}^{+}$ — Pseudo-inverse of matrix $\mathbf{X}$
v_{jk} — Positive Scalar value (weight) for each UE k in cell j.
N — Number of antennas
Uppercase bold letters denote matrices, while lowercase bold letters denote column vectors. Uppercase letters or lowercase letters without boldface are used for scalars. Sets are represented by calligraphic letters.

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